# SYLLABUS FOR M. Sc. COURSE IN MATHEMATICS (CBCS SYSTEM)

2023

Department of Mathematics Diamond Harbour Women's University Sarisha, West Bengal 743368. (Modified and Effective from the Academic Session 2022-23)

# Syllabus for Two-Year Four-Semester PG Course in Mathematics Diamond Harbour Women's University COURSE STRUCTURE

**Grand Total Marks – 1000** 

# **SEMESTER WISE DIVISION OF MARKS AND CREDITS**

SEMESTER	Marks Theoreti	Marks Practical	Discipline Centric Elective		Open Elective	Total Marks	Total Credits	Study hours
	cal (Core)	(Core)	Theory	Practical	(Theory)			
SEMESTER-I	250	-	250			250	20	240
SEMESTER-II	250	-	250			250	20	240
SEMESTER-III	135	15	135	15	100	250	20	240
SEMESTER-IV	250		250			250	20	240
Total	885	15	885	15	100	1000	80	960

# FIRST YEAR (FIRST SEMESTER)

Paper Code:	Paper Name Marks		Marks	Total Marks in	Total Credits	Study hours
		Inte rnal	Semester exam	End semester	End semester	nours
MATH/CC/4101	Real Analysis	10	40	50	4	60
MATH/CC/4102	Complex Analysis	10	40	50	4	60
MATH/CC/4103	Abstract Algebra	10	40	50	4	60
MATH/CC/4104	Linear Algebra	10	40	50	4	60
MATH/CC/4105	Classical Mechanics.	10	40	50	4	60
Total		50	200	250	20	240

# FIRST YEAR (SECOND SEMESTER)

Paper Code:	Paper Name	I	Marks	Total	Total	Study
		Inte rnal	Semester exam	Marks in End semester	Credits End semester	hours
MATH/CC/4201	Topology	10	40	50	4	60
MATH/CC/4202	Functional Analysis	10	40	50	4	60
MATH/CC/4203	Ordinary Differential Equations	10	40	50	4	60
MATH/CC/4204	Partial Differential Equations & Calculus of Variations	10	40	50	4	60
MATH/CC/4205	Integral Equations & Integral Transforms.	10	40	50	4	60
Total		50	200	250	20	240

# SECOND YEAR (THIRD SEMESTER)

Paper Code:	Paper Name	M	Marks		Total	Study
	Ivanie	Inte rnal	Semes ter exam	Marks in End semester	Credits End semester	hours
MATH/CC/5101	Numerical Analysis and Laboratory	10	40	50	4	60
MATH/CC/5102	Discrete Mathematics and Differential Geometry	10	40	50	4	60
MATH/CC/5103	Continuum Mechanics	10	40	50	4	60
MATH/OE/5104	Pure Mathematics	10	40	50	4	60
MATH/OE/5105	Applied Mathematics	10	40	50	4	60
Total		50	200	250	20	240

# SECOND YEAR (FOURTH SEMESTER)

Paper Code:	Paper Name Marks		Iarks	Total	Total	Study
		Inter nal	Semeste r exam	Marks in End semester	Credits End semester	hours
MATH/CC/5201	OPTIONAL*					
		10	40	50	4	60
MATH/CC/5202	OPTIONAL*	10	40	50	4	60
MATH/CC/5203	OPTIONAL*	10	40	50	4	60
MATH/CC/5204	OPTIONAL*	10	40	50	4	60
MATH/CC/5205	Project & Viva.		50	50	4	60
Total		40	220	250	20	240

# Two-Year, 4-Semester PG Course in Mathematics:

# **Grand Total Marks – 1000**

Each 4-credit theory course comprises of 60 contact hours (50L+10T) with full marks of 50 of which 20% is based on internal assessment (I.A) & 80% on end semester examination.

# 1<sup>st</sup> Semester:

MATH/ CC /4101: Real Analysis MATH/CC//4102: Complex Analysis MATH/CC/4103: Abstract Algebra MATH/CC/4104: Linear Algebra MATH/CC/4105: Classical Mechanics.

# 2<sup>nd</sup> Semester:

MATH/CC/4201:	Topology
MATH/CC/4202:	Functional Analysis
MATH/CC/4203:	Ordinary Differential Equations
MATH/CC/4204:	Partial Differential Equations & Calculus of
	Variations
MATH/CC/4205:	Integral Equations & Integral Transforms.

# 3<sup>rd</sup> Semester:

MATH/CC/5101: Numerical Analysis and Laboratory MATH/CC/5102: Discrete Mathematics and Differential Geometry MATH/CC/5103: Continuum Mechanics

**Open Elective Paper (CBCS, for other disciplines)** 

MATH/OE/5104: Pure Mathematics MATH/OE/5105: Applied Mathematics

# 4<sup>th</sup> Semester:

MATH/CC/5201: Optional\* MATH/CC/5202: Optional\* MATH/CC/5203: Optional\* MATH/CC/5204: Optional\* MATH/CC/5205: Project & Viva.

# SEMESTER – I

# MATH/TH/CC/4101: Real Analysis

#### Paper Code: MATH/CC/4101

Credit: 4. Full Marks: 50. Study hours=60

# Fourier Series and Fourier Transformation .

# **Bounded Variation**.

Functions of Bounded Variation and their properties, Differentiation of a function of bounded variation, Absolutely Continuous Function, Representation of an absolutely continuous function

by an integral.

#### The Theory of Measure .

Semiring and ring of sets,  $\sigma$ -ringand  $\sigma$ -algebra, Ring and  $\sigma$ ring generated by a class of sets, Monotone class of sets, Monotone class generated by a ring, Borel Sets. Measures on semirings

and their properties, Outer Measure and Measurable Sets, Caratheodory Extension : Outer measure generated by a measure, Lebesgue measure on R<sub>n</sub>, Measure space, Finite and ofinite measure spaces. Measurable Functions, Sequence of measurable functions, Egorov's Theorem,

Convergence in Measure.

#### The Lebesgue Integral.

Simple and Step Functions, Lebesgue integral of step functions, Upper Functions, Lebesgue integral of upper functions, Lebesgue Integrable functions, Fatou's Lemma, Dominated Convergence Theorem, Monotone Convergence Theorem, Riemann integral as a Lebesgue integral, LebesgueVitali Theorem, Application of the Lebesgue Integral.

#### **References:**

- 1. Aliprantis, C.D., Burkinshaw, O., *Principles of Real Analysis*, 3rd Edition, Harcourt Asia Pte Ltd., 1998.
- 2. Royden, H.L., Real Analysis, 3rd Edition, Macmillan, New York & London, 1988.
- 3. Halmos, P.R., Measure Theory, Van Nostrand, New York, 1950.
- 4. Rudin, W., Real and Complex Analysis, McGrawHill Book Co., 1966.

5. Kolmogorov, A.N., Fomin, S.V., Measures, Lebesgue Integrals, and Hilbert Space,

Academic Press, New York & London, 1961.

Note : This course is based on book (1), Chapters 3, 4.

# MATH/TH/CC/4102: Complex Analysis Credit: 4. Full Marks: 50. Study hours=60

#### **Complex Numbers:**

Complex Plane, Lines and Half Planes in the complex plane, Extended plane and its Spherical

Representation, Stereographic Projection.

#### **Complex Differentiation:**

Derivative of a complex function, Comparison between differentiability in the real and complex

senses, Cauchy Riemann equations, Necessary and Sufficient Criterion for complex

differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

# **Complex Functions and Conformality:**

Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Analytic functions as mappings,

Conformal maps, Möbius Transformations.

# **Complex Integration :**

The complex integral (over piecewise C^1 curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions, Morera's Theorem, Goursat's Theorem, Liouville's Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications,

Schwarz's Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

# Singularities :

Definitions and Classification of singularities of complex functions, Isolated singularities, Laurent series, CasoratiWeierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Argument Principle, Rouche's Theorem.

# **Analytic Continuation :**

Schwarz Reflection Principle, Analytic Continuation along a path, Monodromy Theorem. *References:* 

1. Conway, J.B., *Functions of one complex variable*, Second Edition, Narosa Publishing House.

2. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi, 1994.

3. Ahlfors, L.V., *Complex Analysis*, McGrawHill, 1979. Rudin, W., *Real and Complex Analysis* 

McGrawHill Book Co., 1966.

4. Hille, E., *Analytic Function Theory* (2 vols.), Gonn & Co., 1959. Titchmarsh, E.C., *The Theory of Functions*, Oxford University Press, London.

5. Ponnusamy, S., Foundations of Complex Analysis, Narosa Publishing House, 1997.

Note : This course is based on the books (1) and (2), as described below:

Section (i) : Books (1) & (2), Chapter I. Section (ii) : Book (2), Chapter II.

Section (iii) : Book (1), Chapter III. Section (iv) : Book (2), Chapters VI, VII, IX.

Section (v) : Book (1), Chapter V & Book (2), Chapter VIII. Section (vi) : Book (1), Chapter IX.

# MATH/TH/CC/4103: Abstract Algebra

# Credit: 4. Full Marks: 50. Study hours=60

# Groups (20 Marks)

Homomorphism of groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem, Generalized Cayley's Theorem, Automorphisms, Inner Automorphisms and Automorphism Groups, Cauchy's Theorem, Sylow Theorems and their applications. **Rings (20 Marks)** 

Ideals and Homomorphisms, Quotient Rings, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Divisibility Theory : Euclidean Domain, Principal Ideal Domain, Unique

Factorization Domain, Gauss' Theorem, Polynomial Rings, Irreducibility of Polynomials. **Number Theory (10 Marks) :** 

Algebraic approach to Fermat's Theorem, Euler's Theorem, Wilson's Theorem, Arithmetic Functions, Definitions and Examples, Perfect Numbers, Chinese Remainder Theorem, Primitive

Roots.

References:

- 1. Artin, M., Algebra.
- 2. P. B. Bhattacharya, S.K.Jain & S.R.Nagpaul Basic Abstract Algebra (Cambridge).

3. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.

4. Goldhaber, J.K., Ehrlich, G., Algebra, The Macmillan Company, CollierMacmillan Limited,

London.

- 5. Herstein, I.N., Topics in Abastract Algebra, Wiley Eastern Limited, Hungerford, T.W., Algebra, Springer.
- 6. Hungerford, T.W., Algebra, Springer.
- 7. Jacobson, N., Basic Algebra, I & II, Hindusthan Publishing Corporation, India.
- 8. Lang, S., Algebra.
- 9. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGrawHill

Companies, Inc.

10. Rotman, J.J., The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.

11. Sen, M.K., Ghosh, S. and Mukhopadhyay, P., Topics in Abstract Algebra, Universities Press,

2006.

- 12. Gareth A Jones and J. Mary Jones : Elementary Number Theory, Springer International Edition.
- 13. Neal Koblitz, A Course in Number Theory and Cryptography, SpringerVerlag, 2<sup>nd</sup> Edition.
- 14. D. M. Burton : Elementary Number Theory, Wim C. Brown Publishers Duireque, Lowa, 1989.

# MATH/TH/CC/4104: Linear Algebra Credit: 4. Full Marks: 50. Study hours=60

Vector spaces of finite and infinite dimensions over a field, existence of basis, finite and infinite

dimensional subspaces, sum and direct sum of subspaces, dimension, complementary subspace,

quotient space, matrices and linear transformations, change of basis and similarity, algebra of linear transformations, rank nullity theorem. Dual space, Adjoints of Linear Transformations, Dual Basis. Eigen Values and Eigen Vectors, Characteristic and Minimal Polynomials, Cayley-Hamilton theorem. Inner product space, Cauchy-Schwartz inequality, orthogonal vectors and orthogonal complements, orthonormal sets and orthonormal basis, Bessel's inequality, GramSchmidt orthogonalization method. Spectral Theorem. Canonical forms : similarity of linear transformations, Diagonalization, invariant subspaces, reduction to triangular forms, Nilpotent transformations, Hermitian, Selfadjoint, unitary and orthogonal transformations, Jordan blocks and Jordan forms, Rational Canonical forms, The primary decomposition theorem, cyclic subspaces of annihilators, cyclic decomposition. Billinear and Quadratic forms.

#### References:

- 1. Artin, M., Algebra.
- 2. Friedberg, Insel and Spence, Linear Algebra.
- 3. Halmos, Finite Dimensional Vector Spaces.
- 4. Hoffman and Kunze, Linear Algebra, Prentice Hall.
- 5. Hungerford, T.W., Algebra, Springer.
- 6. Kumerason, S., Linear Algebra.
- 7. Lang, S., Linear Algebra.
- 8. Rao and Bhimsankaran, Linear Algebra.
- 9. Jin Ho Kwak and Sungpyo Hong, Linear Algebra, Birkhauser.

#### MATH/TH/CC/4105: Classical Mechanics Credit: 4. Full Marks: 50. Study hours=60

Laws of Motion: Moving axes. Particle Motion Relative to the Rotating Earth. Foucault's Pendulum. Coriolis Force. Virial Theorem. Two and Three Body Motions. Generalised Coordinates. Unilateral and Bilateral Constraints. Principle of Virtual Work. D'Alembert's Principle. Holonomic and Nonholonomic Systems. Scleronomic and Rheonomic Systems. Lagrange's Equation of Motion. Applications. Energy Equation for Conservative Fields. Cyclic or Ignorable Coordinates. Routh's Equations. Dynamical Systems of Liouville's Type Hamilton's Equations of Motion. Calculus of Variations. Hamilton's Principle. Lagrange's Equations of Motion form Hamilton's Principle. Principle of Least Action. Constants of Motion. Noether's Theorem. Conservation Laws. Infinitesimal transformations. Motion of a Rigid Body about a Fixed Point in it. Euler's Dynamical Equations. Eulerian angles. Gyroscope and nonholonomic Problems. Motion of a Symmetrical Spinning Top on a perfectly Rough Floor. Stability of Steady Precession. Canonical Transformations. Generating Functions. Poisson's Bracket. Jacobi's Identity. Poisson's Theorem. Jacobi-Poisson Theorem. Hamilton-Jacobi Equation. Jacobi's Theorem. Hamilton's Principle Function. Hamilton's Characteristic Function. Action Angle Variables. Adiabatic Invariance. Theory of Small Oscillations (Conservative System). Normal Coordinates. Oscillations under Constraints. Stationary Character of Normal Modes. Elements of Nonlinear Oscillations. Newtonian potential function. Formulation of potential and attraction for volume, surface and linear distributions of matter. Potential of a body at a distant point. Examples. Equipotential sufaces. Potential function at points in free space. Potential and attraction at a point within matter, existence, continuity and differentiability. Behaviour of the first partial derivatives and of logarithmic potential at large distances. An introduction to Quantum Mechanics. **Books Recommended:** 

1. H.Goldstein, Classical Mechanics. Narosa Publishing House, New Delhi, (1980).

2. F.Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow 1975)

3. J.L.Synge and B.A. Griffith, Principles of Mechanics, McGrawHill, N.Y. (1970)

4. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New

Delhi (1998)

5. N.H. Louis and Janet D.Finch, Analytical Mechanics, C.U.P. (1998)

6. E.T. Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P. (1977)

7. S.W. McCusky, An Introduction to Advanced Dynamics, AddisonWesley Publ. Co. Inc.

Massachusetts (1953)

- 8. A. S. Ramsey, Dynamics PartII, C.U.P. (1972)
- 9. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers Moscow
- (1973)

10. R.H. Dicke and J.P. Wittke, Introduction to Quantum Mechanics, Addison Wesley, (1960)

- 11. Rydnik, ABC of Quantum Mechanics, Peace Publisher, Moscow.
- 12. F.Chorlton, Textbook of Fluid Dynamics, CBS Publications, Delhi, 1985.
- 13. A.S. Ramsey Newtonian Attractions. (Cambridge)
- 14. O.D. Kellog Foundations of Potential Theory, Dover (1963).

# MATH/TH/CC/4201: Topology

# Credit: 4. Full Marks: 50. Study hours=60

# Set Theory :

Countable and Uncountable Sets, Schroeder Bernstein Theorem, Cantor's Theorem, Cardinal Numbers and Cardinal Arithmetic, Continuum Hypothesis, Zorn's Lemma, Axiom of Choice, Well Ordered Sets, Maximum Principle, Ordinal Numbers.

# **Topological Spaces and Continuous Functions :**

Topological spaces, Basis and Sub basis for a topology, Order Topology, Product topology on  $X \times Y$ , subspace Topology, Interior Points, Limit Points, Derived Set, Boundary of a set, Closed

Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology, Continuous Functions, Open maps, Closed maps and Homeomorphisms, Product Topology, Quotient Topology, Metric Topology, Complete Metric Spaces, Baire Category Theorem.

# **Connectedness and Compactness :**

Connected and Path Connected Spaces, Connected Sets in R, Components and Path Components, Local Connectedness. Compact Spaces, Compact Sets in R, Compactness in Metric

Spaces, Totally Bounded Spaces, AscoliArzelà Theorem, The Lebesgue Number Lemma, Local

Compactness.

# References:

1. Munkres, J.R., *Topology, A First Course*, Prentice Hall of India Pvt. Ltd., New Delhi,2000.

- 2. Dugundji, J., Topology, Allyn and Bacon, 1966.
- 3. Simmons, G.F., Introduction to Topology and Modern Analysis, McGrawHill, 1963.
- 4. Kelley, J.L., General Topology, Van Nostrand Reinhold Co., New York, 1995.
- 5. Hocking, J., Young, G., *Topology*, AddisonWesley Reading, 1961.
- 6. Steen, L., Seebach, J., *Counter Examples in Topology*, Holt, Reinhart and Winston, New York, 1970.

Note : This course is based on the book (1), Chapters 1 5. 0.5in

# MATH/TH/CC/4202: Functional Analysis

Credit: 4. Full Marks: 50. Study hours=60

# **Banach Spaces :**

Normed Linear Spaces, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Quotient Space of normed linear spaces and its completeness, Riesz Lemma, Fixed Point Theorems and its applications. Bounded Linear Transformations, Normed linear spaces of bounded linear transformations, Uniform Boundedness Theorem, Principle of Condensation of Singularities, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, HahnBanach Theorem, Dual Space, Reflexivity of Banach Spaces.

# **Hilbert Spaces :**

Real Inner Product Spaces and its Complexification, CauchySchwarz Inequality, Parallelogram

law, Pythagorean Theorem, Bessel's Inequality, GramSchmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Structure

of Hilbert Spaces, Orthogonal Complement and Projection Theorem. Riesz Representation Theorem, Adjoint of an Operator on a Hilbert Space, Reflexivity of Hilbert Spaces, Selfadjoint

Operators, Positive Operators, Projection Operators, Normal Operators, Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

# **References:**

1. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte

Ltd., 1998.

2. Goffman, C., Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New

Delhi, 1987.

- 3. Bachman, G., Narici, L., Functional Analysis, Academic Press, 1966.
- 4. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.
- 5. Simmons, G.F., Introduction to Topology and Modern Analysis, McGrawHill, 1963.
- 6. Limaye, B.V., Functional Analysis, Wiley Eastern Ltd.
- 7. Conway, J.B., A Course in Functional Analysis, Springer Verlag, New York, 1990.
- 8. Kreyszig, E., Introductory Functional Analysis and its Applications, John Wiley and Sons, New York, 1978.

# MATH/TH/CC/4203: Ordinary Differential Equations Credit: 4. Full Marks: 50. Study hours=60

First order system of equations: Wellposed problems, existence and uniqueness of the solution.

simple illustrations. Peano's and Picard's theorems (statements only), Linear systems, nonlinear

autonomous system, phase plane analysis, critical points, stability, Linearization, Liapunov stability, undamped pendulum, Applications to biological system and ecological system. **Special Functions** 

Series Solution : Ordinary point and singularity of a second order linear differential equation in

the complex plane; Fuch's theorem, solution about an ordinary point, solution of Hermite

equation as an example; Regular singularity, Frobenius' method – solution about a regular singularity, solutions of hypergeometric, Legendre, Laguerre and Bessel's equation as examples

Legendre polynomial : its generating function; Rodrigue's formula, recurrence relations and differential equations satisfied by it; Its orthogonality, expansion of a function in a series of Legendre Polynomials Adjoint equation of nth order: Lagrange's identity, solution of equation from the solution of its adjoint equation, selfadjoint equation, Green's function *References:* 

1. Codington ,E.A and Levinson , N ., Theory of ordinary differential equation , Mcgraw Hill

2. Estham, Ordinary differential equation.

3. Hartman, P, Ordinary differential equation, John wiley and sons

4. Reid , W.T . Ordinary differential equation , John wiley and sons .

5. Burkhill , J .C ., Theory of ordinary differential equation

6. Ince , E .L . Ordinary differential equation , Dover

# MATH/TH/CC/4204: Partial Differential Equations and Calculus of Variations

Credit: 4. Full Marks: 50. Study hours=60

# Group-A: Theory of Partial Differential Equations (30 marks)

Introduction, Cauchykowalewski's theorem (statement only) classification of second order partial differential equations to Hyperbolic,. Elliptic and Parabolic types. Reduction of linear and

quasilinear equations in two independent variables to their canonical forms, characteristic curves.

Wellposed and illposed problems.

# (i) Hyperbolic Equations:

The equation of vibration of s string. Formulation of mixed initial and boundary value problem.

Existence, uniqueness and continuous dependence of the solution to the initial conditions. D'Alembert's formula for the vibration of an infinite string. The domain of dependence, the domain of influence use of the method of separation of variables for the solution of the problem

of vibration of a string. Investigation of the conditions under which the infinite series solution convergence and represents the solution. Riemann method of solution, Problems, Transmere of

vibration of membrances. Rectangular and circular membranes problems.

# (ii) Elliptic Equations :

Occurrence of Laplace's equation. Fundamental solutions of laplace's equation in two and three independent variables. Laplace equation in polar, Spherical polar and in cylindrical polar coordinates, Minimum – Maximum theorem and its consequences. Boundary value problems, Dirichlets and Neumann's interior and exterior problems uniqueness and continuous dependence of the solution on the boundary conditions. Use of the separation of variables method for the solution of Laplace's equations in two and three dimensions interior and exterior Dirichlet's problem for a circle, and a semi circle, steadystate heat flow equation Problems, Higher dimensional problems, Dirichlet's problem for a cube, cylinder and sphere, Green's function for the Laplace equation, in two and three dimensions.

# Group-B: Calculus of variations. (20 marks)

Variational problems with fixed boundaries Euler's equation for functionals containing First order derivative and one independent variable. Extremals. Functionals dependent on higher

order derivatives. Functionals dependent on more than one independent variable. Variational problems in parametric form. Invariance of Euler's equation under coordinate transformation. Variational problems with moving boundaries. Functionals dependent on one and two functions.

One sided variations. Sufficient conditions for an extremum — Jacobi and Legendre conditions. Second variation. Variational principle of least action. Applications. *References:* 

# 1. Elements of Partial Differential Equations – Ian N. Sneddon (McGraw Hill).

- 2. An Elementary Course in Partial Differential Equations T. Amarnath (Narosa).
- 3. Partial Differential Equations, Graduate Series in Mathematics, Vol.19 L. C. Evans (1998, AMS)
- 4. Partial Differential Equations Miller.
- 5. Partial Differential Equations F. John
- 6. Phoolan Prasad, Renuka Ravindran: Partial Differential Equations.
- 7. John David Logan: Applied Partial Differential Equations.
- 8. Emmanuele Di Benedetto: Partial Differential Equations.
- 9. Andrei Dmitrievich Polianin, Vadim Fedorovich Zaitsev, Alan Moussiaux: Handbook of first Order Partial Differential Equations.
- 10. Tyn Myint U., Lokenath Debnath: Linear Partial Differential Equations for Scientists and Engineers.
- 11. M. Gelfand and S.V. Fomin Calculus of Variations (Prentice Hall 1963)
- 12. A.S. Gupta Calculus of Variations with Applications {Prentice Hall 1997)

# MATH/TH/CC/4205: Integral Equations & Integral Transforms Credit: 4. Full Marks: 50. Study hours=60

# **Group-A: Integral Equations – 25 marks**

1. Linear integral equations of 1st and 2nd kinds – Fredholm and Volterra types. Relation between integral equations and initial boundary value problems .

2. Existence and uniqueness of continuous solutions of Fredholm and Volterra's integral equations of second kind. Solution by the method of successive approximations. Iterated kernels.

3. Fredholm theory for the solution of fredholm's integral equation. Fredholm's determinant D(lamda). Fredholm's first minor D(x,y,lamda) Fredholm's first and second fundamental relations. Fredholm's pth minor. Fredholm's first, second and third fundamental theorems. Fredholm's alternatives.

4. HilbertSchmidt theory of symmetric kernels. Properties of symmetric kernels. Existence of characteristic constants. Complete set of characteristic constants and complete orthonoralmised

system of fundamental functions. Expansion of iterated kernel  $k_{\ell}(x,t)$ , in terms of fundamental

functions. Schmidt's solution of Fredholm's integral equations.

5. Applications

# Group-B: Integral Transforms – 25 marks

1. Fourier Transforms: Fourier integral Theorem. Definition and properties. Fourier transform of the derivative. Derivative of Fourier transform. Fourier transforms of some useful functions. Fourier cosine and sine transforms. Inverse of Fourier transforms. Convolution. Properties of convolution function. Convolution theorem. Applications.

2. Laplace transforms: Definition and properties. Sufficient conditions for the existence of Laplace Transform. Laplace Transform of some elementary functions. Laplace transform of the

derivatives. Inverse of Laplace transform. Bromwich Integral Theorem. Initial and final value theorems. Convolution theorem. Applications.

# Books:

1. S. G. Mikhlin - Integral Equation (Pergamon Press).

- 2. F. G. Tricomi Integral Equation (Interscience Publishers).
- 3. A.Chakrabarti, Applied Integral Equation (Vijay Nicole Imprints Pvt Ltd).
- 4. I.N. Sneddon Use of Integral transform (McGraw Hill).

5. I.N. Sneddon Fourier Transforms (McGraw Hill).

# MATH/THL/CC/5101: Numerical Analysis and Laboratory Credit: 4. Full Marks: 50. Study hours=60

# Numerical Analysis (35 marks)

Numerical Methods : Algorithm and Numerical stability. Graffae's root squaring method and Bairstow's method for the determination of the roots of a real polynomial equation. Polynomial Approximation : Polynomial interpolation; Errors and minimizing errors; Tchebyshev polynomials; Piecewise polynomial approximation. Cubic splines; Best uniform approximations, simple examples. Richardson extrapolation and Romberg's integration method; Gauss' theory of quadrature. Evaluation of singular integral. Operators and their interrelationships : Shift, Forward, Backward, Central differences; Averaging operators, Differential operators and differential coefficients Initial Value Problems for First and Second order O.D.E. by

(i) 4th order R – K method

(ii) RKF 4 method

(iii) Predictor - Corrector method by AdamBashforth,

AdamMoulton and Milne's method. Boundary value and Eigenvalue problems for second order O.D.E. by finite difference method and shooting method. Elliptic, parabolic and hyperbolic P.D.E. (for two independent variables) by finite difference method; Concept of error, convergence & numerical stability.

# References:

- 1. F. B. Hildebrand Introduction to Numerical Analysis
- 2. Demidovitch and Maron Computational Mathematics
- 3. F. Scheid Computers and Programming (Schaum's series)
- 4. G. D. Smith Numerical Solution of Partial Differential Equations (Oxford)
- 5. Jain, Iyengar and Jain Numerical Methods for Scientific and Engineering Computation
- 6. A. Gupta and S. C. Basu Numerical Analysis
- 7. Scarborough Numerical Analysis
- 8. Atkinson Numerical Analysis
- 9. A. Ralston A First Couse in Numerical Analysis, McGrawHill, N.Y. (1965)

# Laboratory: (15 marks)

- 1. Inversion of a nonsingular square matrix.
- 2. Solution of a system of linear equations by Gauss Seidel method.
- 3. Integration by Romberg's method.
- 4. Initial Value problems for first and second order O.D.E. by
- (i) Milne's method (First order)
- (ii) 4th order Runge Kutta method (Second order)
- 5. Dominant Eigen pair of a real matrix by power method (largest and least).
- 6. B.V.P. for second order O.D.E. by finite difference method and Shooting method.
- 7. Parabolic equation (in two variables) by two layer explicit formula and Crank– Nickolson –

implicit formula.

8. Solution of one dimensional wave equation by finite difference method.

# References:

- 1. Xavier, C. C Language and Numerical Methods, (New Age International (P) Ltd. Pub.)
- 2. Gottfried, B. S. Programming with C (TMH).
- 3. Balaguruswamy, E. Programming in ANSI C (TMH).

# MATH/TH/CC/5102: Discrete Mathematics and Differential Geometry Credit: 4. Full Marks: 50. Study hours=60

# **Discrete Mathematics**

Graphs, Vertex degrees, Walks, Paths, Trails, Cycles, Circuits, Subgraphs, Induced subgraph, Cliques, Components, Complete Graphs. Bipartite Graphs.Connected Graphs, Trees, Eulerian Graphs, Hamiltonian Graphs, Traveling Salesman Problem, Planar Graphs, Vertex coloring, Chromatic number.

# **Tensors:**

Tensor and their transformation laws, Tensor algebra, Contraction, Quotient law, Reciprocal tensors, Kronecker delta, Symmetric and skewsymmetric tensors, Metric tensor, Riemannian space, Christoffel symbols and their transformation laws, Covariant differentiation of a tensor, Riemannian curvature tensor and its properties, Bianchi identities, Riccitensor, Scalar curvature, Einstein space.

# **Curves in Space:**

Parametric representation of curves, Helix , Curvilinear coordinates in E 3 . Tangent and first curvature vector, Frenet formulas for curves in space, Frenet formulas for curve in E n. Intrinsic

differentiation, Parallel vector fields, Geodesic.

# References:

1. Introduction to Graph Theory, Douglas B. West, PrenticeHall of India Pvt. Ltd., New Delhi

2003.

- 2. Graph Theory, F. Harary, AddisonWesley, 1969.
- 3. Basic Graph Theory, K.R. Parthasarathi, Tata McGrawHill Publ. Co. Ltd., New Delhi, 1994.
- 4. Graph Theory Applications, L.R. Foulds, Narosa Publishing House, New Delhi, 1993.
- 5. Graph Theory with Applications, J.A. Bondy and U.S.R. Murty, Elsevier science, 1976.
- 6. Graphs and Digraphs, G. Chartrand and L. Lesniak, Chapman & Hall, 1996.
- 7. Graph Theory with Applications to Engineering and Computer Science, NarsinghDeo,

PrenticeHall of India Pvt.Ltd., New Delhi, 1997.

8. RaywardSmith, V. J., A First Course in Formal Language Theory.

9. Davis, M., and Weyuker, E. J., Computability, Complexity, and Languages.

- 10. Hopcroft, and Ullman, Introduction to Automata Theory, Languages, and Computation.
- 11. Tensor Calculas and Application to Geometry and Mechanics : (chapterII and III) I.S.SOKOLNIKOFF.

12. An Introduction to Differential Geometry: (chapter – I,II,III,V and VI)T.T.WILMORE. Differential Geometry: BARY SPAIN

# MATH/TH/CC/5103 : Mechanics of Continuam Credit: 4. Full Marks: 50. Study hours=60

Principles of continuum mechanics, axioms. Forces in a continuum. The idea of internal stress.

Stress tensor. Equations of equilibrium. Symmetry of stress tensor. Stress transformation laws.

Principal stresses and principal axes of stresses. Stress invariants. Stress quadric of Cauchy. Shearing stresses. Mohr's stress circles. Deformation. Strain tensor. Finite strain components in rectangular Cartesian coordinates. Infinitesimal strain components. Geometrical interpretation of infinitesimal strain components.Principal strain and principal axes of strain. Strain invariants. The compatiability conditions. Compatibility of strain components in three dimensions.

Constitutive equations. Inviscid fluid. Circulation. Kelvins energy theorem. Constitutive equation for elastic material and viscous fluid. Navier Stokes equations of motion. Motion of deformable bodies. Lagrangian and Eulerian approaches to the study of motion of continua. Material derivative of a volume integral. Equation of continuity. Equations of motion. Equation of angular momentum. Equation of Energy. Strain energy density function. *References:* 

1. Y.C. Fung : A first course in continuum mechanics.

2. A.C. Eringen : Mechanics of continua.

3. L.I. Sedov : A course in continuum mechanics. Vol – I.

4. W. Prager : Mechanics of continuous media.

# MATH/TH/OE/5104 : Pure Mathematics

Credit: 4. Full Marks: 50. Study hours=60

# 1. Algebra:

Group theory; properties of group, symmetry group, cyclic group, quotient group, group homomorphism and isomorphism, Cauchy's Theorem, Sylow's Theorems. Ring theory; properties of ring, ideal, quotient ring, ring homomorphism, integral domain, Euclidean domain, Unique Factorization domain, polynomial ring and field, Few applications of Group and Ring. Permutations and combinations; Binomial coefficients and Pascal's Triangle. Basic counting principle, The Pigeonhole Principle and its applications.

# 2. Real Analysis:

Real number system and its structure, infimum, supremum, Dedekind cuts. Sequences and series of real numbers, subsequences, monotone sequences, limit inferior, limit superior,

convergence of sequences and series, Cauchy criterion, root and ratio tests for the convergence of series, absolute and conditional convergence. Functions of several variables, directional derivative, partial derivative, total derivative, Jacobian, chain rule and mean value theorems, higher derivatives, interchange of the order of differentiation, Taylor's theorem, extremum problems. Metric spaces, open sets, closed sets, limit points, convergence, completeness, spaces of continuous functions.

# MATH/TH/OE/5105 : Applied Mathematics Credit: 4. Full Marks: 50. Study hours=60

# 1. Numerical Analysis & Partial Differential Equation:

Lagrange's and Newton's interpolation. Bisection, NewtonRapshon methods. Linear equations: Gausselimination method, Gauss-Seidal method. Integration by trapezoidal and Simpson 1/3 methods. Solution of ordinary differential equation by Euler's method, Runge-Kutta methods. Solution of partial differential equation: Finite Element Method.

Classification of second order partial differential equations. Solution of three Fundamental equations: Laplace, Wave and Diffusion Equation. Solution of Neumann and Dirichlet problems, Green's function.

# 2. Fluid Mechanics

Kinematics (equation of Continuity): Lagrangian and Eulerian Method, Stream Line, Boundary Surface, Equation of Continuity and its applications. Equations of Motion: Equation of Motion, Pressure Equation, Bernoulli's Equation, Kelvin's circulation Theorem, equation of energy, d'Alembert's paradox. Two-dimensional Motion: Sources, Sinks and Doublets, Complex potential, Blasius Theorem. Waves: wave motion, Stationary waves and progressive waves, Energy of waves, Waves at the common surface of two liquid, Group velocities, Applications. Vortex Motion: Properties of vortex filaments, Image of vortex w.r.t. plane and circular cylinder, Kirchhoff vortex theorem, Karman's vortex sheet.